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RADIATION STRUCTURE OF THE RELAXATION ZONE OF A SHOCK WAVE IN TWO-PHASE RAREFIED MEDIA

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Methods are presented for calculating the radiation structure of a shock front in a two-phase medium with low density and an example of such a calculation is given.

The problem of the role of radiation in gasdynamics of high-temperature two-phase media is always encountered when the radiation energy flux becomes comparable or exceeds the gasdynamic flux. Thus, if the equilibrium temperature behind the shock front propagating in such media is $\sim 10^3$ K, then radiation has a large effect on the structure of the front with densities in front of the wave less than 10^{-2} kg/m^3. The investigation of the radiation structure of the front of a shock wave in gas in the diffusion approximation involves an analysis of the properties of the transfer equation at $\pm\infty$ and joining the functions sought at the density jump [1, 2]. In this case, when radiation transfer occurs in a background of other relaxation processes, the analysis of the properties at $\pm\infty$ and the numerical solution of the problem as a whole, as noted in [3] and as will be evident in what follows, involve serious difficulties.

The radiation structure of a shock wave in a gas with particles was investigated previously in [4], wherein the interaction of retardation processes, heat exchange with the gas, and radiation of particles without taking into account their effect on the gas flow was determined. In this paper, the processes indicated are examined in a higher-order approximation and, in addition, their interaction with the gasdynamics is determined and two methods are proposed for calculating the radiation structure of the shock wave.

In what follows, we investigate the stationary structure of a shock front in a mixture of a gas and microscopic particles. It is assumed that all particles are spherical and have the same radius, and the gas and the particles are characterized by different temperatures and mass velocities. Phase transformations of the particles are not examined. The gas is assumed to be nonradiating and does not react with the particles, and it is also assumed to be nonviscous and nonthermally conducting. The effects of viscosity and thermal conductivity are taken into account only in the interaction between the gas and the particles.

The structure of the shock front is characterized by two regions of flow, separated by a density jump. In the region behind the density jump, the particles on interacting with the gas exchange heat and mechanical energy, and also loose or acquire energy through radiation. These processes, interacting with one another, determine the structure of the shock front in a given region. The radiation, leaving the surface of the discontinuity, is absorbed in front of the discontinuity by particles, which are thereby heated. In their turn, the particles heat the gas in front of the discontinuity, creating a pressure gradient in it, which puts the gas, and then the particles, into motion. At low density of the two-phase medium, the radiation can greatly change the

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temperature and velocity of particles as well as the value of gasdynamic parameters before their shock compression.

We will examine a one-dimensional stationary two-phase flow in a system of coordinates fixed to the density discontinuity. We orient the x axis along the flow. Taking into account the basic assumptions of the two-velocity and two-temperature model of two-phase flow, the laws of conservation of mass, momentum, and energy in the radiating medium and the equation of state of the gas have the form [1, 5, 6]:

$$\rho v = M,\ \rho_p v_p = M_p,\ \rho v^2 + \rho_p v_p^2 + p = \mathscr{P},$$
$$M\left(\frac{v^2}{2} + C_p T\right) + M_p\left(\frac{v_p^2}{2} + C_\mathrm{m} T_p\right) + S = E, \tag{1}$$
$$p = A\rho T.$$

The equation of motion of the particle and the equation determining the change in its temperature have the form

$$\frac{4\pi}{3} R^3 \rho_\mathrm{m} v_p \frac{dv_p}{dx} = F,$$
$$\frac{dT_p}{dx} = \frac{Q}{\frac{4\pi}{3} R^3 \rho_\mathrm{m} C_\mathrm{m} v_p} - \frac{1}{M_p C_\mathrm{m}} \frac{dS}{dx}. \tag{2}$$

Since in what follows we examine particles whose radius does not exceed the mean free path in a gas, in order to determine F and Q we will make use of the results of rarefied-gasdynamics [7, 8]:

$$F = \frac{\pi R^2 \rho V (v - v_p)}{2} \left\{\frac{(2 - \alpha_n + \alpha_\tau)}{2z^3} \times \right.$$
$$\left. \times \left[\frac{4z^4 + 4z^2 - 1}{2z} \operatorname{erf}(z) + \frac{2z^2 + 1}{\sqrt{\pi}} \exp(-z^2)\right] + \frac{2\alpha_n}{3z}\sqrt{\frac{\pi T_p}{T}}\right\},$$
$$Q = 4\pi R^2 \rho V A \alpha_e (T_r - T_p) \frac{\gamma + 1}{\gamma - 1} \mathrm{St}.$$

Here,

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)\, dt$$

is the probability integral;

$$\mathrm{St} = \frac{1}{8z^2}\left[\frac{z \exp(-z^2)}{\sqrt{\pi}} + \left(z^2 + \frac{1}{2}\right)\operatorname{erf}(z)\right]$$

is the modified Stanton number;

$$T_r = T\,[1 + z^2 r (\gamma - 1)/(\gamma + 1)]$$

is the stagnation temperature;

$$r = 2 + \frac{1}{z^2} - \frac{\operatorname{erf}(z)}{8z^4\,\mathrm{St}}$$

is the modified stagnation coefficient.

In what follows, we assume that the absorption coefficient of the particles does not depend on frequency (grey body approximation). We will write the transfer equation for the radiation flux S and density U in the diffusion approximation:

$$\frac{dS}{dx} = c\varkappa\,[B(x) - U],\quad \frac{dU}{dx} = -\frac{3\varkappa}{c} S. \tag{3}$$

Here $\varkappa$ is the coefficient of absorption of the particles, equal to the product of the number of particles per unit volume n_p by their cross section πR^2 and the emissivity θ,

$$\varkappa = \theta n_p \pi R^2 = \frac{3\,\theta}{4\,R\rho}\,\rho_p\,;$$

$B(x) = 4\sigma T_p^4/c$ is the source function in the transfer equation.

The system of equations (1)-(3) is valid on both sides of the density discontinuity. On the discontinuity itself, there are no sources and sinks, so that all variables S, U, T_p, and v_p in the system (2)-(3) at $x = 0$ are continuous. At $\pm\infty$, they take their equilibrium values and are determined by the value of the four parameters. For these parameters, we choose the particle and gas density at $-\infty$ and the temperature of the mixture at $\pm\infty$. Denoting the quantity relating to the region in front of the density discontinuity by the index 1, we write the boundary conditions for the system of differential equations at $\pm\infty$ and the continuity equation at the density discontinuity as follows:

$$S_1 = 0,\ U_1 = \frac{4\,\sigma T_{-\infty}^4}{c},\ T_{p1} = T_{-\infty},\ v_{p1} = v_{-\infty},\ x = -\infty, \tag{4a}$$

$$S_1 = S,\ U_1 = U,\ T_{p1} = T_p,\ v_{p1} = v_p,\ x = 0, \tag{4b}$$

$$S = 0,\ U = \frac{4\,\sigma T_\infty^4}{c},\ T_p = T_\infty,\ v_p = v_\infty,\ x = +\infty. \tag{4c}$$

In the region in front of the density discontinuity, the point $x = -\infty$ is a singular point of the system (1)-(3). It can be shown, e.g., that in the SU plane of the multidimensional space S, U, T_p, and V_p, this point is a saddle point, while one of the separatrices is the solution sought. In order to determine the equation of the separatrix, we will linearize the system of differential equations, expanding it near the singular point with respect to small deviations in the variables from their equilibrium values, i.e., according to S, $\Delta U = U - U_{-\infty}$, $\Delta T_p = T_p - T_{-\infty}$, and $\Delta v_p = v_p - v_{-\infty}$. This leads to a system of linear homogeneous differential equations with constant coefficients for $S, \Delta U, \Delta T_p, \Delta v_p$. Solving it using Euler's method, we set

$$S = \gamma_1 e^{\lambda x},\ \Delta U = \gamma_2 e^{\lambda x},\ \Delta T_p = \gamma_3 e^{\lambda x},\ \Delta v_p = \gamma_4 e^{\lambda x}.$$

The characteristic equation of the system, a fourth-degree algebraic equation, has (as can be shown) a single positive and three negative roots. In order to satisfy the boundary conditions at $-\infty$, the arbitrary constants in front of the exponentials with negative roots must be set equal to zero. Denoting the positive root as λ_+, and the arbitrary constant associated with it by C_1, we write the particular solution of the system (1)-(3) in the linear approximation in the form:

$$S = C_1 e^{\lambda x},\ \Delta U = \beta_U C_1 e^{\lambda x} = \beta_U S,\ \Delta T_p = \beta_T C_1 e^{\lambda x} = \beta_T S, \tag{5}$$
$$\Delta v_p = \beta_v C_1 e^{\lambda x} = \beta_v S,$$

where $\lambda = \lambda_+$; $x \leqslant 0$; β_U, β_T, and β_v are expressed in terms of the coefficients of the linearized system. Relations (5) permit integrating the starting nonlinear system (1)-(3) numerically. Given some small value of the flux S, it is possible to obtain from (5) the values of the remaining variables corresponding to it, with the exception of the variable x. Its uncertainty is a result of the uncertainty in the position of the front, which can be eliminated with the help of the continuity conditions for the solution on each side of the density jump, strictly tied to the reference system for x. The integration was continued using the usual Runge–Kutta scheme until the value of U became so large that the solution obtained obviously encompasses the entire region in front of the density jump. (In the problem examined, U in front of the jump does not exceed $U_{+\infty}$).

In the region behind the density jump, the use of the boundary conditions, obtained from the conditions at $+\infty$ using the scheme presented above, does not give desirable results. The point is that in this region the characteristic equation of the linearized system (1)-(3) has, as also in the region in front of the density jump, three negative and a single positive root, so that in the region examined (where $x \geq 0$), from conditions (4c), it is possible to determine only a single arbitrary constant. If further boundary conditions found are used for obtaining the solution sought, satisfying conditions (4b), then in accordance with the number of undetermined arbitrary constants in the region behind the shock front it will be necessary, in carrying out the numerical integration, to construct an infinity of solutions in a cube.

The qualitative difference in the solutions in front of the density jump and behind it is related to the fact that in the region in front of the jump all processes are determined by absorption of radiation transferred out of the region $x \geq 0$. In order to determine the flux and density of radiation in this region, besides the conditions at $-\infty$ for S and U, it is necessary to fix one of these quantities on the density jump. The conditions indicated, together with the rates of the corresponding processes, completely determine the value of the temperature and velocity of particles in the entire region examined. The presence of a single undetermined constant in (5) is related to this circumstance. In order to integrate the system of differential equations (1)-(3) in the region behind the density jump, on the jump itself it is necessary to fix in addition to S or U the temperature and velocity of the particles. In this region, heat-exchange processes and the resistance of particles are relatively independent. Although they depend on the transfer of energy by radiation, but are not determined only by such transfer, as in the region in front of the density jump, they can occur in the absence of radiation. The presence of three undetermined constants in the solution of the system (1)-(3), obtained in the linear approximation, is related to the necessity of giving the three quantities on the density jump as boundary conditions.

The boundary conditions on the density jump can be obtained with the help of the continuity conditions (4b) from the solutions of the system (1)-(3) relating to the region before and after the density jump. In its turn, the solution behind the density jump is determined by the boundary conditions on the jump. In order to avoid the difficulties indicated, we will transform from the system of differential equations (3) to a system of integral equations. We introduce a new independent variable ξ with the help of equation $d\xi = \sqrt{3}\varkappa dx$ and boundary conditions $\xi = 0$ at $x = 0$. The transformed equations (3) form a system of linear inhomogeneous differential equations. Solving this system using the method of variation of an arbitrary constant and eliminating one of two integration constants with the help of the conditions (4c), we obtain the following system

$$S = \frac{c}{\sqrt{3}}\left\{C_2 \exp(-\xi) + \frac{1}{2}\int_0^{\xi} \exp[-(\xi - t)]\, B(t)\, dt - \frac{1}{2}\int_{\xi}^{\infty} \exp[-(t-\xi)]\, B(t)\, dt\right\}, \tag{3a}$$

$$U = C_2 \exp(-\xi) + \frac{1}{2}\int_0^{\infty} \exp[-|t-\xi|]\, B(t)\, dt.$$

In the region behind the density jump, the system of integrodifferential equations (1), (2), and (3a) is solved by iteration. In the zeroth-order approximation, it is assumed that there is no radiation. The boundary conditions for heat exchange processes and retardation of particles are the velocity and temperature of the particles corresponding to some value of the coordinate of the solution of the system (1)-(3) in the region in front of the density jump. First, the system (1)-(2) is integrated together with the equations $d\xi = \sqrt{3}\varkappa dx$ and the zeroth-order approximation for S and U, for chosen boundary conditions for the variables T_p and v_p and with the condition $\xi = 0$ at $x = 0$. According to the functions $T_p(x)$ and $v_p(x)$, we calculate successively the integrals in the system (3a), the arbitrary constant C_2 with the help of the conditions (4b), the solution in front of the density jump, and the value of the radiation flux S(x) and density U(x) in the first approximation. Then, the cycle described above is repeated with the same boundary conditions, using now not the zeroth but the first, second, etc. order approximations for S and U to obtain S(0) and U(0) to given accuracy. Then, the temperature $T_p(0)$ and particle velocity $v_p(0)$ are determined from the solution obtained in the region in front of the density jump according to the corresponding values of S(0) and U(0). Together with the functions S(x) and U(x) obtained in the last iteration in the preceding cycle they form the initial data for the new cycle. The calculation is terminated after the required accuracy is obtained for T_p and v_p. As a result, the solution S(x), U(x), $T_p(x)$, and $v_p(x)$ is obtained for regions behind and in front of the density jump, satisfying all conditions (4).

The algorithm for solving the system of equations (1), (2), and (3a) described above differs somewhat from a similar algorithm described in [3] and extends, in comparison to [3], the region of convergence of the iterations to small flux densities. Up to flux densities at $-\infty$ not lower than 10^{-2} kg/m^3, the convergence of the iterations is good: an accuracy of 0.1% is attained with not more than 10 iterations. As the flux density decreases and the role of radiation correspondingly increases, the convergence of the process indicated worsens. When the structure of the shock front behind the density jump is determined mainly by radiation transfer, the iteration process no longer converges.

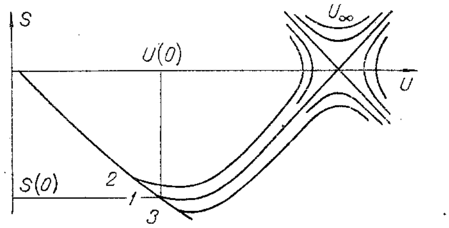

Fig. 1. Qualitative behavior of the solutions of the system (1)-(3) in the SU plane ($S = 0$, $U = U_\infty$ is the saddle point singularity; S(0), U(0) are the values of S and U on the density jump): 1) the solution for which the boundary coincides with the density jump; 2 and 3) are located, respectively, in front of and behind the jump.

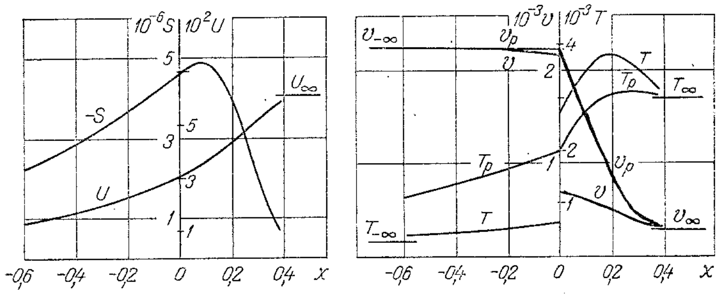

Fig. 2. Profiles of the radiation flux S [J/m^2 · sec], radiation density U [J/m^3], gas velocity v and particles velocity v_p [m/sec], gas temperature T, and particle temperature T_p [K]; x is in meters.

The system of differential equations (1)-(3) can be integrated in the region behind the density jump with the help of another method, which is also effective for low gas densities. The gist of the method is as follows. In the solution of the system (1)-(3) obtained for the region in front of the density jump with the position of the jump remaining undetermined, some point is chosen with radiation density $U_C^0 = (U_A^0 + U_D^0)/2$, where U_A^0 is the maximum value of U up to which the integration was carried out; U_D^0 is the value of U known to correspond to some point in front of the density jump sought ($U_D^0 = 4\sigma T_{-\infty}^4/c$ can be taken as this point). The quantity U_C^0 and the corresponding value of the remaining variables of the solution obtained are used as boundary conditions in integrating the system (1)-(3) behind the density jump. The position of U_C^0 relative to the density jump sought is determined by the behavior of the integral curves in the neighborhood of the singular point $x = +\infty$. It is assumed that the singular point of the system of differential equations (1)-(3) in the SU plane is a saddle point. (The existence of a saddle point singularity for the system (3) with $T_p = T$ is proved rigorously in [2].) The nature of the behavior of the solution of (1)-(3) sought, which must coincide with the separatrix as well as other integral curves in the vicinity of the saddle point singularity, is shown in Fig. 1. Curve 1 represents the case when the boundary coincides with the value of the variables on the density jump, while curves 2 and 3 correspond to the case when they are located, respectively, in front of and behind density jump. By comparing the solution obtained with curves 1-3, it is easy to determine the direction in which U_C^0 must be displaced in order to approach the integral curve sought. Solving this problem by iteration, it is sufficient, e.g., to set $U_C^i = (U_C^{i-1} + U_M^{i-1})/2$, where for curve 2 $U_A^i = U_C^{i-1}$, $U_D^i = U_D^{i-1}$, $U_M^{i-1} = U_D^{i-1}$; for curve 3, $U_A^i = U_A^{i-1}$, $U_D^i = U_C^{i-1}$, and $U_M^{i-1} = U_A^{i-1}$, where i is the number of the iteration. It is easy to see that the density jump is always located within the segment $[U_A^i, U_D^i]$, whose length from iteration to iteration decreases by a factor of 2. The number of iterations for given accuracy is determined by the convergence of the process of halving the segments and the initial length of $[U_A^0, U_D^0]$. Calculations using this method showed that for gas densities of interest for the given problem the values of the variables sought on the density jump and the entire structure of the shock front are determined to within 1% within ≈10 iterations. The equilibrium values at the point $x = +\infty$ are achieved in this case by all variables with much higher accuracy. The latter circumstance served as a criterion for the correctness of the calculation of the structure of the shock front and an indirect proof of the fact that the point $x = +\infty$ is a saddle point singularity of the system of differential equations (1)-(3) in the SU plane.

The procedure described above can be realized when the mean free path of the radiation exceeds the characteristic length of the other nonequilibrium processes (which, as a rule, is observed in this problem) or when the gas density is low, while radiation does not play a large role. Otherwise, the radiation attains a state of local thermodynamic equilibrium sooner than the other relaxation processes are completed. Then, for boundary conditions given imprecisely on the density jump, the integral curves of the radiation flux and density will be above or below local equilibrium. In this case, in determining U_C^i, the computational procedure must first ensure that S and U attain locally equilibrium values and then equilibrium values.

Figure 2 shows an example of the structure of a shock front in a mixture of neon and soot particles, corresponding to the following parameters of the problem: $\rho = 1.34 \cdot 10^{-3}$ kg/m^3; $\rho_p = 1.34^{-3}$ kg/m^3; T = 273°K at $x = -\infty$; T = 3000°K at $x = +\infty$; $R = 5 \cdot 10^{-7}$ m. In the calculations, it was assumed that $\alpha_e = \alpha_n = \alpha_\tau = 0.9$; $\theta = 1$; $C_m = 7.68 \cdot 10^4$ J/kg·deg. As is evident from Fig. 2, radiation absorbed in front of the density jump heats the particles and gas to a very high temperature and changes the velocity of the particles and that of the gas. Due to the continuous increase in T_p and ρ_p on the density jump, the particles absorb more radiation than they emit, not only for $x < 0$, but also in some neighborhood $x \geq 0$. For this reason, the point of maximum radiation flux does not coincide with the density jump. The structure behind the density jump is determined by the interaction of all nonequilibrium processes. Immediately behind the jump, the determining process is the process of conversion of the kinetic energy of particles into heat energy in the mixture. This explains the increase in the gas temperature behind the density jump.

NOTATION

A, C_p, and C_m, respectively, gas constant, the heat capacity of the gas and of the particle material computed per unit mass; B(x), source function in the transfer equation; c, velocity of light; C_1, and C_2, arbitrary constants; F, force acting on particle in the gas flow; M, M_p, P, and E, constants that characterize the state of the gas in front of the shock wave, the intensity of the shock wave, the fraction of particles and gas in the mixture; n_p, number of particles per unit volume; p, gas pressure; Q, amount of heat absorbed per unit time by a particle moving in the gas; r, modified stagnation coefficient; R, particle radius; S, integral radiation flux; St, Stanton number; t, integration variable; T and T_p, gas and particle temperatures; T_r, stagnation temperature; U, integral radiation density; v and v_p, gas and particle velocities; $V = |v - v_p|$; x, distance measured from the density jump; $z = V/\sqrt{2AT}$; α_e, α_n, and α_τ, accommodation coefficients, characterizing the fraction of the energy and of the normal and tangential components of the momentum transmitted to the gas atoms in collisions with a solid surface, respectively; β_U, β_T, and β_V, constant coefficients in the linearized system (1)-(3); γ, ratio of specific heat capacities of the gas; γ_1, γ_2, γ_3, γ_4, and λ, constants; θ, emissivity; $\varkappa$, coefficient of absorption; λ_+, positive root of the characteristic equation; ξ, dimensionless independent variable of the system (1)-(3); ρ, ρ_p, and ρ_m, gas density, particle density, and density of the particle material; σ, Stefan – Boltzmann constant. The indices are as follows: A, D, and C, respectively, edge and midpoints of a segment; i, number of the iteration; $-\infty$, equilibrium value of the variables in front of the shock front; $+\infty$, equilibrium value of the variables behind the shock front.

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